Field theory model giving rise to "quintessential inflation" without the cosmological constant and other fine-tuning problems

A. B. Kaganovich*
Physics Department, Ben Gurion University of the Negev, Beer Sheva 84105, Israel
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A field theory is developed based on the idea that the effective action of a yet unknown fundamental theory, at an energy scale below the Planck mass M_p, has the form of expansion in two measures: $S=\int d^4x[\Phi L_1 + \sqrt{-g}L_2]$, where the new measure Φ is defined using the antisymmetric tensor field $\Phi d^4x=\partial_{[\alpha}A_{\beta\gamma\delta]}dx^\alpha \wedge dx^\beta \wedge dx^\gamma \wedge dx^\delta$. A shift $L_1 \rightarrow L_1+\text{const}$ does not affect the equations of motion, whereas a similar shift when implementing with L_2 causes a change which in standard GR would be equivalent to that of the cosmological constant (CC) term. The next basic conjecture is that the Lagrangian densities L_1 and L_2 do not depend on $A_{\mu\nu\lambda}$. The new measure degrees of freedom result in the scalar field $\chi=\Phi/\sqrt{-g}$ alone. A constraint appears that determines χ in terms of matter fields. After the conformal transformation to the new variables (Einstein frame), all equations of motion take the canonical GR form of the equations for gravity and matter fields and, therefore, the models we study are free of the well-known defects that distinguish the Brans-Dicke type theories from Einstein's GR. All novelty is revealed only in an unusual structure of the effective potentials and interactions which turn over our intuitive ideas based on our experience in field theory. For example, the greater Λ we admit in L_2, the smaller magnitude of the effective inflaton potential $U(\phi)$ will there be in the Einstein picture. Field theory models are suggested with explicitly broken global continuous symmetry, which in the Einstein frame has the form $\phi \rightarrow \phi+\text{const}$. The symmetry restoration occurs as $\phi \rightarrow \infty$. A few models are presented where the effective potential $U(\phi)$ is produced with the following shape: for $\phi \lesssim -M_p$, $U(\phi)$ has the form typical for inflation model, e.g., $U=\lambda\phi^4$ with $\lambda \sim 10^{-14}$; for $\phi \gtrsim -M_p$, $U(\phi)$ has mainly the exponential form $U \sim e^{-a\phi/M_p}$ with variable *a*; $a=14$ for $-M_p \gtrsim \phi \lesssim M_p$, which gives the possibility for nucleosynthesis and large-scale structure formation; and $a=2$ for $\phi \gtrsim M_p$, which implies the quintessence era. There is no need for any fine-tuning to prevent the appearance of the CC term or any other terms that could violate the flatness of $U(\phi)$ at $\phi \gg M_p$. $\lambda \sim 10^{-14}$ is obtained without fine-tuning as well. Quantized matter field models, including spontaneously broken gauge theories, can be incorporated without altering the results mentioned above. Direct coupling of fermions to the inflaton resembles Wetterich's model, but there is a possibility to avoid any observable effect at the late universe. SSB does not raise any problems with the CC in the late universe.

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I. INTRODUCTION

Recent high-redshift and cosmic microwave background data [1] suggests that a small effective cosmological constant gives a dominant contribution to the energy density of the present universe. Among the attempts to describe this picture, the idea to profit by the properties of a slow-rolling scalar field (quintessence model) [2–8] seems to be the most attractive and successful. In such an approach, the present vacuum energy density $\rho_{vac} \sim 10^{-47}$ GeV^4 has to be imitated by the energy density of a slowly-rolling scalar field down its potential $U(\phi)$, which presumably approaches zero as $\phi \rightarrow \infty$. However, all known quintessence models contain two fundamental problems.

(1) The cosmological constant problem [9,10] remains in the quintessence models as well. Particle physics and cosmology must give a distinct mechanism that enforces the effective cosmological constant to decay from an extremely large value in the very early universe to an extremely small present value without a fine-tuning of parameters and initial conditions.

(2) All known quintessence models are based on the choice of some specific form for the potential $U(\phi)$. The general feature of the potentials needed to realize quintessence is that $U(\phi)$ must be flat enough, as ϕ is large enough, in order to provide conditions for the slow-roll approximation. However, it is not clear what happens with other possible terms in the potential, including quantum corrections (see Kolda and Lyth, [11]). In fact, the potential may, for instance, contain terms that constitute a structure of polynomials in ϕ (and $\phi^n \ln \phi$), and they are not negligible as ϕ is large, unless an extreme fine tuning is assumed for the mass and self-couplings. For example, the restriction of the flatness conditions on the quartic self-interaction $\lambda\phi^4$ [11] is $\lambda \ll 10^{-120}(M_p/\phi)^2$.

In this paper, I present a field theory model that resolves the above fine tuning problems and besides that, this model is able to give a broad range of tools for constructive answers for a few more important questions.

(3) In the framework of a model where the potential $U(\phi)$ of the exponential or inverse power low (or their combinations [8]) form plays the role of a quintessential potential as ϕ is large enough, the question arises as to what is the cosmological role of $U(\phi)$ as ϕ is close to zero or negative?

*Email address: alexk@bgumail.bgu.ac.il

If some other scalar field is responsible for an inflation of the early universe, then a field theory has to explain why the potential $U(\phi)$ of the scalar field ϕ is negligible as ϕ is close to zero or negative. However, if the same quintessence field ϕ also plays the role of the inflaton [12,13] (in the early universe), then a field theory again has to explain [14] an origin of the relevant effective potential. Of course, this is a nontrivial problem. For example, Peebles and Vilenkin [13] have presented an interesting model of a single scalar field that drives the inflation of the early universe and ends up as quintessence. They adopt the monotonic potential

$$U(\phi)=\lambda m^4[1+(\phi/m)^4] \quad \text{for} \quad \phi<0,$$

$$=\frac{\lambda m^4}{1+(\phi/m)^\alpha} \quad \text{for} \quad \phi\geqslant 0, \tag{1}$$

where $\alpha=\text{const}>0$ (for example, 4 or 6) and the parameters $\lambda=10^{-14}$ and $m=8\times 10^5$ GeV were adjusted in [13] to achieve a satisfactory agreement with the main observational constraints. It is well known [15] that such an extremely small value of λ is dictated in the $\lambda\phi^4$ theory of the chaotic inflation scenario by the necessity to obtain a density perturbation $\delta\rho/\rho\sim 10^{-5}$ in the observable part of the universe. In other words, the potential of this quintessential inflation model includes both the fine tuning required by the inflation of the early universe and the fine-tuning dictated by the quintessence model of the late universe. As it is pointed out in Ref. [13], it seems also to be an unnatural feature of this model that a small mass $m=8\times 10^5$ GeV$\ll M_p$ must appear in the potential of the inflaton field ϕ interacting only with gravity. And finally, one should apparently believe that such a quintessential inflation potential must be generated by some field theory without fine tuning. These problems are typical for the quintessential inflation type models [12,13] .

(4) It is well known that the coincidence problem [16] can be avoided in the framework of the quintessence models that make use of tracker potentials [8]. The exponential potential with $a=\text{const}$,

$$U(\phi)=U_0 e^{-a\phi/M_p}, \tag{2}$$

is a special example of a tracker solution [8]. In spacially flat models with such potential, the ratio of the scalar field ϕ energy density to the total matter energy density rapidly approaches a constant value determined by a and the matter equation of state [2,3,7] (see also Ref. [17], where a similar result was achieved in the context of Kaluza-Klein-Casimir cosmology). However the strong constraint on Ω_ϕ dictated by cosmological nucleosynthesis ($\Omega_\phi\lesssim 0.2$) [6,7,18] predetermines the ϕ fraction to remain a subdominant one in the future that apparently contradicts the observable accelerated expansion. A possible resolution of this problem proposed by Wetterich [6] consists of the idea that a in Eq. (2) might be ϕ dependent. In that case, it would again be very attractive to develop a field theory model where the exponential potential, Eq. (2), with an appropriate ϕ-dependent a, is generated in a natural way.

(5) Since the mass of excitations of the ϕ field has to be extremely small in the present-day universe ($m_\phi\lesssim H_0 \sim 10^{-33}$ eV), possible direct couplings of ϕ to the standard matter fields should give rise to very long-range forces which do not obey the equivalence principle [19]. To prevent such undesirable effects, the very strong upper limits on the coupling constants of the quintessence field to the standard matter fields have to be accepted without any known reason. An attempt to construct a model where an unbroken symmetry could support zero mass of ϕ excitations [20] inevitably runs against the necessity to start from a trivial potential [19]; without knowledge of a mechanism for the breaking of this symmetry, such small coupling constants may be introduced into a theory only by hand.

It will be shown in this paper that one can answer all the above questions in the framework of the field theory model based on the hypothesis that the effective action of the fundamental theory at the energy scales below the Planck mass can be represented in a general form including two measures and, respectively, two Lagrangian densities

$$S=\int [\Phi L_1+\sqrt{-g}L_2]d^4x. \tag{3}$$

Here, $\sqrt{-g}$ is the standard measure of integration in the action principle of both Einstein's general relativity (GR) and other gravitational theories making use of general coordinate invariance. The measure Φ is defined using the antisymmetric tensor field $A_{\mu\nu\lambda}$,

$$\Phi d^4x=\partial_{[\alpha}A_{\beta\gamma\delta]}dx^\alpha\wedge dx^\beta\wedge dx^\gamma\wedge dx^\delta, \tag{4}$$

and Eq. (3) is also invariant under general coordinate transformations. Notice that the measure Φ is a total derivative and, therefore, a shift $L_1\to L_1+\text{const}$ does not affect the equations of motion, whereas a similar shift when implementing L_2 causes a change which, in standard GR, would be equivalent to that of the cosmological constant term. The next basic conjecture is that the Lagrangian densities L_1 and L_2 do not depend on $A_{\mu\nu\lambda}$. In this paper, I refer to this theory as the two measures theory (TMT).

The main features of TMT have been studied in series of papers [21–26].

II. SOME GENERAL FEATURES OF TMT

Let us consider a simple model with the scalar field ϕ:

$$S=\int d^4x\left[\Phi\left(-\frac{1}{\kappa}R(\Gamma,g)+\frac{1}{2}g^{\mu\nu}\phi_{,\mu}\phi_{,\nu}-V_1(\phi)\right)+\sqrt{-g}V_2(\phi)\right]. \tag{5}$$

The case where $V_2(\phi)\equiv\text{const}$ was studied in Ref. [24], and the general case was studied by Guendelman in Ref. [25]. TMT gives desirable results if we proceed in the first order formalism (metric $g_{\mu\nu}$ and connection $\Gamma^\mu_{\lambda\sigma}$ are independent variables, as well as the antisymmetric tensor field $A_{\mu\nu\lambda}$), and $R(\Gamma,g)=g^{\mu\nu}R_{\mu\nu}(\Gamma)$, $R_{\mu\nu}(\Gamma)=R^\alpha_{\mu\nu\alpha}(\Gamma)$, and

$$R^{\lambda}_{\mu\nu\sigma}(\Gamma)\equiv\Gamma^{\lambda}_{\mu\nu,\sigma}+\Gamma^{\lambda}_{\alpha\sigma}\Gamma^{\alpha}_{\mu\nu}-(\nu\leftrightarrow\sigma). \tag{6}$$

At this stage no specific forms for $V_1(\phi)$ and $V_2(\phi)$ are assumed.

Variation of the action with respect to $A_{\mu\nu\lambda}$ results in the equation $\epsilon^{\mu\nu\alpha\beta}\partial_\beta L_1=0$, which means that

$$L_1=-\frac{1}{\kappa}R(\Gamma,g)+\frac{1}{2}g^{\mu\nu}\phi_{,\mu}\phi_{,\nu}-V_1(\phi)=sM^4=\text{const}, \tag{7}$$

where sM^4 is an integration constant, $s=\pm1$, and M is a constant of the dimension of mass.

Variation with respect to $g^{\mu\nu}$ leads to

$$-\frac{1}{\kappa}R_{\mu\nu}(\Gamma)+\frac{1}{2}\phi_{,\mu}\phi_{,\nu}-\frac{1}{2\chi}V_2(\phi)g_{\mu\nu}=0, \tag{8}$$

where the scalar field χ is defined by

$$\chi\equiv\frac{\Phi}{\sqrt{-g}}. \tag{9}$$

The consistency condition of Eqs. (7) and (8) takes the form of the constraint

$$V_1(\phi)+sM^4-\frac{2V_2(\phi)}{\chi}=0. \tag{10}$$

We have defined χ and Φ to be of the same sign. To avoid problems which could appear if the measure Φ becomes singular $(\Phi=0)$, in what follows, we must care about such choices of V_1, V_2, and sM^4 that the constraint (10) provides for χ to be positive definite. Then Φ will be positive definite as well. If for example, $V_2(\phi)$ is positive definite, then $V_1(\phi)+sM^4$ must be non-negative.

Solution of equations obtained by variation of the action with respect to $\Gamma^{\mu}_{\lambda\sigma}$ can be represented (see [22–24]) as a sum of Christoffel's connection coefficients $\{^{\lambda}_{\mu\nu}\}$ of the metric $g_{\mu\nu}$, and a non-Riemannian part which is a linear combination of $\sigma,_\mu$ where $\sigma\equiv\ln\chi$.

The scalar field ϕ equation is

$$(-g)^{-1/2}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu\phi)+\sigma,_\mu\phi^{,\mu}+\frac{dV_1}{d\phi}-\frac{1}{\chi}\frac{dV_2}{d\phi}=0. \tag{11}$$

In the conformal frame defined by the conformal transformation

$$g_{\mu\nu}(x)\rightarrow g'_{\mu\nu}(x)=\chi g_{\mu\nu}(x),\quad \phi\rightarrow\phi,\quad A_{\mu\nu\lambda}\rightarrow A_{\mu\nu\lambda}, \tag{12}$$

the non-Riemannian contribution into the connection disappears: $\Gamma^{\lambda}_{\mu\nu}\rightarrow\overline{\Gamma}^{\lambda}_{\mu\nu}=\overline{\{^{\lambda}_{\mu\nu}\}}$ (here, $\overline{\{^{\lambda}_{\mu\nu}\}}$ are Christoffel's connection coefficients of the Riemannian space-time with the metric $g'_{\mu\nu}$). Tensors $R^{\lambda}_{\mu\nu\sigma}(\Gamma)$ and $R_{\mu\nu}(\Gamma)$ transform into the Riemann $R^{\lambda}_{\mu\nu\sigma}(g'_{\alpha\beta})$ and Ricci $R_{\mu\nu}(g'_{\alpha\beta})$ tensors, respectively, in the Riemannian space-time with the metric $g'_{\mu\nu}$. After making use of the solution for χ as it follows from the constraint, Eq. (10), the gravitational equations (8) and the scalar field equation (11) in the new conformal frame obtain the standard form of the Einstein's GR equations for the self-consistent system of gravity $(g'_{\mu\nu})$ and scalar field ϕ with the TMT effective potential (for details, see [24–26])

$$U(\phi)=\frac{1}{\chi^2}V_2(\phi)=\frac{1}{4V_2(\phi)}[sM^4+V_1(\phi)]^2. \tag{13}$$

Notice that just $U(\phi)$ plays the role of the true potential that governs the dynamics of the scalar field ϕ, while $V_1(\phi)$ and $V_2(\phi)$ have no sense of the potential energy densities themselves, but rather, they generate the potential energy density. This is why we will use the term *prepotentials* for $V_1(\phi)$ and $V_2(\phi)$. Notice that our choice of the sign in front of the prepotential $V_2(\phi)$ is opposite to the usual one that would be in the case of the standard GR. Doing this is just for convenience in what follows.

In order to provide a disappearance of the cosmological constant, one usually demands that the effective potential be equal to zero at the minimum, i.e., it is necessary that the effective potential and its first derivative are equal to zero at the same point. As a matter of fact, this is the essence of the cosmological constant problem treated in the old sense, when there was no need for an explanation of a small but nonzero cosmological constant. If we want to avoid the necessity to fulfill this fine tuning, TMT gives us such an opportunity (it has been explored in Refs. [24–26]). In fact, independently of the shape of the nontrivial prepotential $V_1(\phi)$, an infinite number of initial conditions exists for which $V_1+sM^4=0$ at some value $\phi=\phi_0$. If $V_1(\phi)$ and $V_2(\phi)$ are regular at $\phi=\phi_0$, and $V'_1(\phi_0)\neq0$ and $V_2(\phi)$ are positive definite, then $\phi=\phi_0$ is the absolute minimum of $U(\phi)$ with the value $U(\phi_0)=0$. We will refer to such a situation as *the first class scenario*.

In the present paper we will study the models with such a prepotential V_1 that there will be an infinite number of initial conditions for which $V_1+sM^4\neq0$ at any value of ϕ (we will refer to such a situation as *the second class scenario*). Then the stable vacuum may, for instance, be realized asymptotically as $\phi\rightarrow\infty$, which is actually the idea used in the quintessence models.

The assumption that $V_2(\phi)$ is positive definite will be our choice in what follows.

III. EXTREMELY BROAD CLASS OF TMT MODELS DO NOT REQUIRE FINE-TUNING TO PROVIDE QUINTESSENCE

A. General idea: The inverse power low quintessential potential as a simple example

In contrast to standard gravitational theories where the quintessential potential must be a slowly decreasing function as $\phi\rightarrow\infty$, in TMT we have an absolutely new option: the quintessential behavior of the TMT effective potential $U(\phi)$ for large enough ϕ may be achieved with increasing prepotentials $V_1(\phi)$ and $V_2(\phi)$. This circumstance enables us to avoid both the cosmological constant problem and the problem of the flatness of the quintessential potential.

For illustration of these statements, let us notice that starting from the positive power low prepotentials V_1 and V_2

$$V_1 = m_1^{(4-n_1)}\phi^{n_1}, \quad V_2 = \tfrac{1}{4} m_2^{(4-2n_2)}\phi^{2n_2}, \tag{14}$$

with $n_2 > n_1$, we obtain the TMT effective potential, which for large enough ϕ, has the inverse power low form

$$U \approx \frac{m_1^{2(4-n_1)}}{m_2^{2(2-n_2)}} \frac{1}{\phi^{2(n_2-n_1)}} \tag{15}$$

and does not depend on the integration constant. Another interesting case is $V_1 \equiv 0$ (remind that adding a constant to V_1 is equivalent just to a redefinition of the integration constant sM^4) and, for example, $V_2 \equiv \lambda\phi^4$. Then $U(\phi) = M^8/\lambda\phi^4$.

Although there exists a possibility for generation of a negative power low potential in the models with dynamical supersymmetry breaking (see, for example [29]), such potential still looks to be exotic in the context of the standard field theory. As we see in TMT, such quintessential forms of the effective potential are obtained very easily and in a natural way.

Besides, adding any subleading (as $\phi \to \infty$) terms to Eqs. (14) does not alter the above results since their relative contributions to $U(\phi)$ will be suppressed as ϕ is large enough. In particular, adding the term $V_2^{(0)}\int\sqrt{-g}d^4x$, $V_2^{(0)} \equiv \text{const}$, which in GR would have the sense of the cosmological constant term, does not affect $U(\phi)$ as ϕ is large enough. Thus, starting from the polynomial form of the prepotentials V_1 and V_2 with an appropriate choice of the powers n_1 and n_2 of the leading terms, one can in fact provide a generation of the inverse power low quintessential potential in such a way that neither the cosmological constant problem nor the the problem of the flatness of the quintessential potential appears at all.

B. The exponential form of the TMT effective potential $U(\phi)$

A simple way to realize an exponential asymptotic form of the TMT effective potential $U(\phi)$, Eq. (13), is to define the prepotentials V_1 and V_2 as follows:

$$V_1 = s_1 m_1^4 e^{\alpha\phi/M_p}, \quad V_2 = \tfrac{1}{4} m_2^4 e^{2\beta\phi/M_p}. \tag{16}$$

Here $s_1 = \pm 1$ and we assume that α and β are positive constants. The restrictions formulated after Eq. (10) have to be taken into account. The effective TMT potential corresponding to the prepotentials (16)

$$U = \frac{1}{m_2^4}(s_1 m_1^4 e^{-(\beta-\alpha)\phi/M_p} + sM^4 e^{-\beta\phi/M_p})^2 \tag{17}$$

contains two particular cases of special interest.

a. The case $\alpha = \beta$. This case corresponds to a sort of the scale invariant theory studied by Guendelman [25]. In fact, in this case the theory, Eq. (5), is invariant under global transformations

$$g_{\mu\nu} \to e^\theta g_{\mu\nu}, \quad A_{\mu\nu\lambda} \to e^\theta A_{\mu\nu\lambda}, \tag{18}$$

whereas the scalar field ϕ undergoes the shift

$$\phi \to \phi - \frac{M_p}{\beta}\theta. \tag{19}$$

In such a model, the TMT effective potential has the form

$$U(\phi) = \frac{m_1^8}{m_2^4}\left[1 + \frac{s_1}{s}\left(\frac{M}{m_1}\right)^4 e^{-\beta\phi/M_p}\right]^2, \tag{20}$$

and the observation that $U(\phi)$ has an infinite flat region as $\phi \to \infty$ and approaches a nonzero constant m_1^8/m_2^4, has been used in Ref. [25] for discussion of possible cosmological applications with the choice $s_1/s = -1$. The first possibility is related to the very early universe: a slow rolling (new inflationary) scenario might be realized assuming that the universe starts at a sufficiently large value of ϕ. Another scenario discussed by Guendelman in Ref. [25] is based on a possibility for m_1^8/m_2^4 to be very small. This approach has the aim to construct a scenario for the very late universe. In this scenario, there could be a long lived stage with almost constant energy density m_1^8/m_2^4 that will eventually disappear when the universe achieves its true vacuum state with zero cosmological constants. This occurs when the expression in parenthesis in Eq. (20) becomes zero and, therefore, no fine-tuning is needed. It turns out (see Refs. [25,26]) that in the presence of a matter, which is introduced in a way respecting the global symmetry Eqs. (18),(19), the change of the constraint (10) leads to a correlation between $U(\phi)$ (close but not equal to zero) and the matter energy density.

In the case $\alpha = \beta$, the TMT effective potential, Eq. (20), is not a constant due to the appearance of a nonzero integration constant M, that is actually due to a spontaneous breaking of the global continuous symmetry, Eqs. (18),(19). Guendelman noticed [25] that in terms of the dynamical variables used in the Einstein frame, that is, $g'_{\mu\nu}$ and ϕ, the symmetry transformations, Eqs. (18),(19), are reduced to shifts (19) alone [$g'_{\mu\nu}$ is invariant under transformations, Eqs. (18),(19)]. Thus in terms of the dynamical variables of the Einstein frame, the spontaneous symmetry breaking is just that of the global continuous symmetry $\phi \to \phi - (M_p/\beta)\theta$. It is important that, as it was mentioned in Ref. [25], this global continuous symmetry is restored as $\phi \to \infty$.

b. The case $\beta > \alpha > 0$. This is the most interesting case from the viewpoint of the quintessence. For $\beta\phi \gg M_p$, the TMT effective potential, Eq. (17), behaves as a decaying exponent:

$$U \simeq \frac{m_1^8}{m_2^4} e^{-2(\beta-\alpha)\phi/M_p} \quad \text{as} \quad \beta\phi \gg M_p. \tag{21}$$

If we want to achieve the quintessential form of the TMT effective potential (21) for not too large a value of ϕ and with not too big a difference in the orders of m_1 and M (this point will be explained in the next section), then we need the condition

$$0<\beta-\alpha\ll\beta. \tag{22}$$

And of course, the most evident argument in favor of this condition consists in the demand to provide the flatness of the ϕ potential at the late ϕ-dominated universe, where it has to imitate the present cosmological constant. This is possible only if $\beta-\alpha$ is less than or of order one, while there are no reasons for β not to be large in general.

Comparing this condition for α and β with that of the model of Ref. [25] discussed just above, one can observe that the model under consideration can be interpreted as that with a small explicit violation of the global symmetry, Eq. (18),(19). Notice that the expression for $U(\phi)$ as $\beta\phi\gg M_p$ does not include the integration constant M and the exponent is proportional to $\beta-\alpha$. This reflects the fact that such asymptotic behavior of $U(\phi)$ results from the explicit violation of the global continuous symmetry, Eqs. (18),(19).

It is very interesting that although the discussed *global continuous symmetry* (18),(19) is broken in this model explicitly, the equations of motion show that the symmetry *is also restored as* $\phi\to\infty$, just as in the case $\alpha=\beta$ with only spontaneous symmetry breaking. Therefore, in terms of the dynamical variables used in the Einstein frame, that is, $g'_{\mu\nu}$ and ϕ, in the model where the condition (22) holds, the approximate global symmetry $\phi\to\phi-(M_p/\beta)\theta$ is restored as $\phi\to\infty$.

This observation opens an unexpected chance to solve the problem discussed by Carroll [19] (problem five in the list of problems in the Introduction), which consists of the following. There are no reasons to ignore a possibility that the scalar field ϕ interacts directly with usual matter fields. Suppose that such interactions have the form of the coupling $f_i\cdot(\phi/m)\mathcal{L}_i$, where $\mathcal{L}_i$ is any gauge invariant dimension-four operator, m is a mass scale, and f_i is a dimensionless coupling constant. The flatness of the quintessential potential of the field ϕ means that excitations of ϕ are almost massless. Therefore, in the presence of direct interactions of the ϕ field to the usual matter fields, one has to expect the appearance of the very long-range forces which do not obey the equivalence principle. Observational restrictions on such fifth forces impose small upper limits on the coupling constants f_i.

To explain the smallness of f_i's, Carroll proposed that the theory possesses an approximate global continuous symmetry of the form $\phi\to\phi+\text{const}$ (the idea similar to what is used in pseudo-Goldstone boson models of quintessence [5] where, however, an explicit breaking of the continuous chiral symmetry reduces it to a discrete symmetry). In the framework of Einstein's GR, such exact continuous symmetry is incompatible with a nontrivial potential of the scalar field ϕ. This means that if we were working in Einstein's GR, and then started from the model with the exact symmetry $\phi\to\phi+\text{const}$ and therefore with a constant potential, we would want to achieve a nontrivial, quintessential potential (passed also across a fine-tuning purgatory) as a result of some mechanism for symmetry breaking. Such a picture looks even more problematic than the fine-tuning problem itself. In addition, in the framework of such a general idea about a breaking of the symmetry $\phi\to\phi+\text{const}$, it is impossible to point out the parameters of the theory which could produce, after a symmetry breaking, the small coupling constants f_i.

In contrast to GR, in TMT one can suppose that in a yet unknown more fundamental theory, the global continuous symmetry, Eqs. (18),(19) is an exact one, and that $\alpha=\beta$. At energies below the Plank mass, the symmetry is breaking and it is assumed that the effective action describing the relevant physics has the form of TMT, Eq. (5) (inclusion of the usual matter will be studied in Sec. VI), with the nontrivial prepotentials, Eq. (16). The only thing we need from a mechanism for symmetry breaking consists of a small relative shift of the magnitudes of α and β satisfying the condition (22). If the symmetry breaking generates couplings of the scalar field ϕ to the usual matter fields, then the corresponding dimensionless coupling constants f_i must be proportional[1] to some positive power of $(\beta-\alpha)/\beta$.

Notice that an unbounded increase of the prepotentials as $\phi\to\infty$ does not produce problems, at least on the classical level, since as was already mentioned in Sec. II, the prepotentials have no sense of a potential energy density. The real potential is the TMT effective potential, Eq. (13), that in the model under consideration, approaches zero according to Eq. (21) as $\phi\to\infty$.

An evident generalization of the prepotentials (16) that maintains the behavior of $U(\phi)$ as $\beta\phi\gg M_p$, Eq. (21), consists of adding to them the terms with a lower degree of growth. They may be, for example, polynomials in ϕ (as was the case in the previous subsection). Relative contributions of all adding terms into the TMT effective potential $U(\phi)$ will be exponentially suppressed for large ϕ. If these additional terms appear as a result of breaking of the symmetry, Eqs. (18),(19) (remind that $\alpha=\beta$ in the case of the exact symmetry), then coefficients in front of them have to be proportional to some positive power of the small parameter $(\beta-\alpha)/\beta$. The latter will be used in the next section. For the same reasons as it was before, the symmetry (18),(19) is restored as $\phi\to\infty$.

Simple reasoning adduced here, as well as in the previous subsection, does not look like a trivial one if we recall that in GR, adding any constant and/or increasing (as $\phi\to\infty$) term to the potential destined to be a quintessential one causes a drastic violation of its desirable features; an arbitrary cosmological constant appears and/or the flatness conditions are destroyed if no extreme fine-tunings are made. The basis for

[1] Notice that the exponents in the prepotentials, Eq. (16), actually contain the dimensional factors αM_p^{-1} and βM_p^{-1}. Therefore, the dimensionless parameter that could characterize the symmetry breaking has to be of the form $(\beta M_p^{-1}-\alpha M_p^{-1})/\beta M_p^{-1}=(\beta-\alpha)/\beta$.

the resolution of these problems in TMT consists in a possibility to achieve a quintessence form of the effective potential as ϕ is large enough, starting from prepotentials increasing as $\phi\rightarrow\infty$. As a matter of fact, this is the main advantage of the studied TMT models over the quintessence models formulated in the framework of the standard GR.

In conclusion, it is worthwhile to notice for the following discussion that in all cases considered in this section, χ^{-1} as the solution of the constraint (10) asymptotically approaches zero as $\phi\rightarrow\infty$.

IV. PROBE MODELS: TOWARDS EFFECTIVE TMT POTENTIAL OF THE QUINTESSENTIAL INFLATION TYPE

A. Some clarifications to the rest of the paper

The previous sections served a preparatory role in the formulation and solution of the main problems of this paper. In Sec. III, our attention was concentrated on the possibilities of TMT to generate without fine-tuning the scalar field ϕ potential which, for large enough ϕ provides a quintessence. It turns out, however, that some of such TMT effective potentials can also be well defined as driving the early universe evolution. In this paper, I do not aim to look for a precise value for all parameters of the potential that could be able to provide an adequate description of the cosmological evolution from slightly after Planck time up to now and answer all demands of the realistic cosmology. But I do want to exhibit the fact that the field theory models based on TMT provide the existence of a broad spectrum of tools giving us the firm belief that such a potential can be generated without fine-tuning. More precisely, in this section I am going to demonstrate that exploring the results of the previous section can make one sure that TMT is able to generate (without any sort of fine-tuning) the effective potential of such a form that could answer basic demands of the realistic cosmology.

Such a qualitative examination is enough for the purposes of this paper, which consists mainly in studying of some basic field-theoretic problems of TMT that turn out to be in very close interrelation with some fundamental features of the cosmological scenario. The essence of the matter is that, generally speaking, the price for the success of TMT in the resolution of the cosmological constant problem is serious enough. In fact, in order to incorporate the matter fields into the simplifying picture reviewed in Sec. II in such a way that the TMT effective equations of motion of all fields in the Einstein frame would have the form of the equations of motion of the standard field theory based on GR in (Ref. [24]), we were forced to start from the very nonlinear (in the matter fields) original TMT action. This circumstance together with the non-Riemannian nature of the original action makes the quantization of TMT practically an inaccessible problem even on a semiclassical level. Moreover, it was unclear how one can approach a question of matter fields quantization in the background curved space-time. We will see below that for the second-class cosmological scenarios (see the end of Sec. II), with an appropriate choice of the prepotentials, it is enough to start from the original TMT action with exactly the same degree of nonlinearity in matter fields as in the standard theory in order to achieve the standard matter field theory in a background (pseudo-Riemannian) space-time. This appears to be possible to do, after the so-called TMT gravitational background is defined in Sec. V. Then the matter fields quantization in the TMT gravitational background reduces to the standard procedure of the matter fields quantization in curved space-time [27,28]. Fortunately, it turns out that the choice of the initial cosmological conditions and prepotentials needed to provide such successful construction of the matter field theory in the context of TMT, corresponds to the class of models where the TMT effective potential allows to solve all five problems mentioned in Introduction.

B. Models based on the hypothesis that the theory possesses the explicitly broken global symmetry

The prepotentials of the form (16) with additional (subleading as $\phi\rightarrow\infty$) terms provide the possibility to generate the TMT effective potential $U(\phi)$ with an asymptotic quintessence behavior that mimics the current effective cosmological constant. For this to be done there is no need for any sort of fine-tuning, and the satisfactory condition for this is $0<\beta-\alpha\ll\beta$ in Eq. (16). If, however, one wants to extend the range of applicability of the TMT effective potential of the same single scalar field ϕ to satisfy constraints of the realistic cosmology from inflation of the early universe up to the present-day universe, then we have too big of an arbitrariness in the choice of the additional terms to Eq. (16). I restrict myself by models based on the idea that the action, Eq. (5), is the effective one of a more fundamental theory at the energy scales below the Planck mass. It seems, then, to be natural to suppose that transition from the fundamental theory to the effective one is accompanied by the breaking of some fundamental symmetries. I will assume that one of such symmetries is the global one, Eqs. (18),(19).[2] Such an approach to the choice of prepotentials enables us to narrow the amount of suitable versions. In particular, for models leading to the asymptotic (as $\phi\rightarrow\infty$) inverse power low TMT effective potentials (discussed in Sec. III A), one cannot point out a range where the symmetry (18),(19) is restored. This is why I am obliged to restrict myself to studying models of the type discussed in Sec. III B and, more precisely, to models where the condition Eq. (22) holds.

Below we will formulate three models where the modifications of the prepotentials, Eq. (16), will be realized by adding the simplest terms explicitly breaking the symmetry (18),(19). The Planck mass M_p is chosen as the typical scale for parameters of the dimension of mass corresponding to the limit where the global symmetry (18),(19) is unbroken. Then the appearance of the mass parameters smaller than M_p is a manifestation of a symmetry breaking by the appropriate terms since those parameters can be represented as $((\beta-\alpha)/\beta)^n M_p$, $n>0$. In the framework of such an approach, one can maintain that the model is free of fine-tuning if orders of all such mass parameters are not too different from

[2]Of course, without knowledge of the fundamental theory, one cannot discuss a mechanism for the symmetry breaking.

M_p [in this connection, see also discussions after Eqs. (21),(22), and footnote 1].

1. Model 1

$$V_1(\phi)=m_1^4 e^{\alpha\phi/M_p}, \quad V_2(\phi)=\tfrac{1}{4}(4V_2^{(0)}+m_2^4 e^{2\beta\phi/M_p}). \tag{23}$$

With the choice of the parameters $m_2=M_p$, $4V_2^{(0)}=(10^{-3}M_p)^4$, $m_1=10^{-2}M_p$, $\beta=7$, and $\alpha=6$, and with the integration constant $M^4=(3q\times 10^{-2}M_p)^4$, $0<q\lesssim 1$ $(s=+1)$, the TMT effective potential $U(\phi)$, Eq. (13), is a monotonically decreasing function with a shape that is convenient to describe in a piecewise form with the following four typical regions:

$$\begin{aligned} U(\phi)&\approx q^8 M_p^4 \quad \text{for} \quad \phi<-2.2M_p,\\ &\approx \frac{q^8 M_p^4}{1+10^{12}e^{14\phi/M_p}} \quad \text{for} \quad -2.2M_p<\phi<-1.8M_p,\\ &\approx 10^{-12}M_p^4 e^{-14\phi/M_p} \quad \text{for} \quad -1.8M_p<\phi<0.6M_p,\\ &\approx 10^{-16}M_p^4 e^{-2\phi/M_p} \quad \text{for} \quad \phi>1.2M_p. \end{aligned} \tag{24}$$

2. Model 2

$$V_1(\phi)=\tfrac{1}{2}\mu_1^2\phi^2+m_1^4 e^{\alpha\phi/M_p},$$

$$V_2(\phi)=\tfrac{1}{4}(4V_2^{(0)}+m_2^4 e^{2\beta\phi/M_p}). \tag{25}$$

With the choice of the parameters $m_2=M_p$, $4V_2^{(0)}=(\tfrac{1}{3}M_p)^4$, $\mu_1=10^{-4}M_p$, $m_1=10^{-3}M_p$, $\beta=7$, and $\alpha=6$ and with the integration constant $M^4=(1/\sqrt{3}10^{-2}M_p)^4$, $(s=+1)$, the TMT effective potential $U(\phi)$, Eq. (13), is a monotonically decreasing function with a shape that one can describe in a piecewise form with the following three typical regions:

$$\begin{aligned} U&\approx \tfrac{1}{4}\lambda\phi^4, \quad \lambda=10^{-14} \quad \text{for} \quad \phi<-\tfrac{1}{3}M_p,\\ &\approx 10^{-16}M_p^4\left[10^{-1}+\frac{1}{2}\left(\frac{\phi}{M_p}\right)^2\right]^2\\ &\quad\times e^{-14\phi/M_p} \quad \text{for} \quad 0<\phi<1.1M_p,\\ &\approx 6\times 10^{-28}M_p^4 e^{-2\phi/M_p} \quad \text{for} \quad \phi>1.4M_p, \end{aligned} \tag{26}$$

where in the interval $0<\phi<1.1M_p$ the factor in front of the exponential function varies very slowly.

3. Model 3

$$V_1(\phi)=\tfrac{1}{2}\mu_1^2\phi^2+m_1^4 e^{\alpha\phi/M_p},$$

$$V_2(\phi)=\tfrac{1}{4}(4V_2^{(0)}+\tfrac{1}{2}\mu_2^2\phi^2+m_2^4 e^{2\beta\phi/M_p}). \tag{27}$$

With the choice of the parameters $m_2=M_p$, $4V_2^{(0)}=(10^{-1}M_p)^4$, $\mu_1=10^{-4}M_p$, $m_1=10^{-3}M_p$, $\mu_2=10^{-2}M_p$, $\beta=7$, and $\alpha=6$, and with the integration constant $M^4=(1/\sqrt{3}10^{-2}M_p)^4$, $(s=+1)$, the TMT effective potential, Eq. (13), is a monotonically decreasing function with a shape that one can describe in a piecewise form with the following three typical regions:

$$\begin{aligned} U&\approx \tfrac{1}{2}m^2\phi^2, \quad m=10^{-6}M_p \quad \text{for} \quad \phi<-0.7M_p,\\ &\approx 10^{-16}M_p^4\left[10^{-1}+\frac{1}{2}\left(\frac{\phi}{M_p}\right)^2\right]^2\\ &\quad\times e^{-14\phi/M_p} \quad \text{for} \quad -0.6<\phi<1.5M_p,\\ &\approx 10^{-24}M_p^4 e^{-2\phi/M_p} \quad \text{for} \quad \phi>1.7M_p, \end{aligned} \tag{28}$$

where in the interval $-0.6M_p<\phi<1.5M_p$, the factor in front of the exponential function varies very slowly.

C. Some general features of the models 1–3

As it was already noted, the exact fitting of all parameters to satisfy the requirements of the realistic cosmology is over and above the plan of this paper. Our aim here is, rather, a demonstration of the extremely broad spectrum of tools given by TMT to solve some fundamental problems of the realistic cosmology.

(1) In each of the models 1–3 with the action (5), the global continuous symmetry (18),(19) is violated by all terms of V_1 and V_2 except for the last term of V_2. The symmetry is restored at the limit $\phi\rightarrow\infty$. All mass parameters (including mass parameters corresponding to Λ terms in each of the models) have orders equal or slightly less than the Planck mass [but not less than the grand unified theory (GUT) scale].

(2) One can see that the TMT effective potential $U(\phi)$ of each of the models 1–3 has a region that can be responsible for an inflation of the early universe. Let us refer to this region of $U(\phi)$ as the inflationary region of U.

In model 1, the inflationary region of U is the infinite interval $-\infty<\phi<-1.8M_p$ with a practically constant value $U(\phi)\approx q^8 M_p^4$ that smoothly passes on a slowly decreasing region. Such an inflationary region of U might be responsible for an initial stage of a new inflationary scenario [30].

In models 2 and 3, the inflationary regions of U have the form of the power low potentials ($\tfrac{1}{4}\lambda\phi^4$ and $\tfrac{1}{2}m^2\phi^2$, respectively) driving the chaotic inflation [15]. Parameters of the prepotentials are chosen in such a way that the inflationary region of U satisfies the requirements of the realistic cosmology. It is very important to stress that this can be done without strong tuning of the parameters, in contrast with the GR approach to the chaotic inflation models, where the strong enough tuning is needed. The choice of β and α does not practically affect the inflationary region of $U(\phi)$.

(3) The TMT effective potential $U(\phi)$ of each of the models 1–3 behaves as

$$U(\phi)\approx \frac{m_1^8}{M_p^4} e^{-2(\beta-\alpha)\phi/M_p} \quad \text{as} \quad \phi>\phi_b=\varpi M_p, \tag{29}$$

where the constant factor ϖ of order one is very sensitive to the choice of parameters. Let us refer to this region of $U(\phi)$ as the quintessential region since it can serve for the quintessential model of the present universe. I should make an important remark here. The quintessential region of U has the form (29) where the value of $(\beta-\alpha)/\beta \ll 1$ determines the strength of the symmetry breaking. The choice of $\beta-\alpha=1$ and $\beta=7$ in models 1–3 has just an illustrative aim, and it is not a problem to adjust the value of $\beta-\alpha$ to satisfy the observable value of Ω_ϕ at present.

(4) Between the inflationary and quintessential regions, there exists an intermediate region of great interest. The TMT effective potential $U(\phi)$ in the intermediate region can be represented in the general form

$$U(\phi)=f(\phi)M_p^4 e^{-2\beta\phi/M_p}, \tag{30}$$

where $f(\phi)$ is a very slowly varying function compared to the exponential factor. There is a remarkable property of the intermediate region of U that provides possibilities for resolution of some of the fundamental problems of the realistic cosmology: by an appropriate choice of β, one can achieve a very rapid decreasing of $U(\phi)$ after an inflationary epoch that provides conditions for transition to the radiation and matter dominated era. For instance, in model 3, the TMT effective potential $U(\phi)$ at the end of the intermediate region ($\phi\approx 1.5M_p$) is roughly 10^{28} times less than at the beginning of the intermediate region ($\phi\approx -0.6M_p$). This property of the intermediate region of U may be very useful in the resolution of problems of cosmological nucleosynthesis constraints and large-scale structure formation [6,7,18,8]. The exact shape of the intermediate region of U (steepness and the range of definition) dictated by the realistic cosmology can be adjusted by the choice of the magnitudes of α, β, and the dimensional parameters (like M, μ_1, etc.).

(5) Combining the intermediate and quintessential regions, one can see that the post-inflation region of the TMT effective potential can be represented approximately in the exponential form described by Eq. (2) with ϕ-dependent parameter a (see Ref. [6])

$$\begin{aligned} a=a(\phi)&=2\beta \quad \text{as} \quad \phi<\phi_b, \\ &=2(\beta-\alpha) \quad \text{as} \quad \phi>\phi_b, \end{aligned} \tag{31}$$

where ϕ_b [see Eq. (29)] is a boundary value of ϕ between the intermediate and quintessential regions of $U(\phi)$. It seems to be very attractive for this result to be obtained in a natural way in the framework of the field theory model without any assumptions specially intended for this. The only things that have been assumed is that the model possesses the approximate global continuous symmetry, Eqs. (18),(19), and the value of $\beta-\alpha \ll \beta$, depending on a strength of the symmetry breaking, should not be large in order to provide the flatness of the TMT effective potential in the quintessential region.

(6) It turns out that in models 2 and 3, the shape of $U(\phi)$ in the buffer range between the inflationary and intermediate regions can be very sensitive to variations of the parameters entering into V_1 and V_2. By means of a suitable change of the parameters one can achieve (without altering the qualitative properties of the discussed above regions), for instance an almost flat shape of U in this buffer range or even successive local minimum and maximum immediately after the inflationary region. This feature of the models may be very important if, for example, one wants to realize a scenario where the instant preheating [31] occurs before entry into the intermediate region.

The final remarks concern the terminology. Since the scalar field ϕ, in the context of models 1–3, dominates both the very early and the late universe, acting in such a way that the universe expands with acceleration, let us call it the inflaton field following the terminology by Peebles and Vilenkin [13].

V. GRAVITATIONAL BACKGROUND AND INFLATON FIELD ϕ

The complicated structure of TMT revealed in Refs. [21–24] turns a quantization procedure into a very serious issue. This concerns the problem of a quantization of all TMT degrees of freedom. But even the matter fields quantization in TMT seems to be a very nontrivial problem. Some reasons for this were discussed briefly in Sec. IV A.

Let us consider here the simple model of Sec. II, where the scalar field ϕ is the only matter field. The system of equations of the standard GR form for gravity and scalar field ϕ with the potential Eq. (13) in the Einstein frame has been obtained for the selfconsistent TMT problem, Eq. (5). At first glance, one can follow a standard procedure when ignoring the reaction of the quantum field fluctuations on the gravity, regarding the scalar field equation as one in the gravitational background that consists of the metric $g'_{\mu\nu}$ treated as an external field (for a short time, I will refer to this as the formal gravitational background). Then the scalar field quantization in such a background would be a well studied problem.

To elucidate the situation, it is useful to look at a possible definition of a gravitational background in TMT before conformal transformations (12), i.e., in the original frame. Namely, let us try to define a gravitational background in the same simple model of TMT starting from the original action, Eq. (5). We are dealing now with two measures: Φ, defined by Eq. (4), and $\sqrt{-g}$. Besides, TMT is formulated in the first-order formalism. Therefore, to determine *the gravitational background in the original frame*, we have to fix the metric $g_{\mu\nu}$, the measure Φ, by means of the antisymmetric tensor field $A_{\mu\nu\lambda}$ and the connection. All geometrical objects that constitute the physically admissible gravitational background have to be self-consistent as the geometrical sector of the complete self-consistent gravity + matter system. This means that the solution mentioned in Sec. II for connection

$\Gamma^{\alpha}_{\mu\nu}$ in terms of $g_{\mu\nu}$ and $\sigma_{,\mu}\equiv(\ln\chi)_{,\mu}$ has to hold.[3] Hence, the gravitational background in the original frame is defined by two fields: the metric $g_{\mu\nu}$ and the scalar field χ. We will refer to this as *the TMT gravitational background* in the original frame. The fact that the scalar field χ has to be regarded as a fixed external field is the origin of problems with the construction of the TMT gravitational background. Let us discuss the reasons for this.

The constraint, Eq. (10), does not include the Newton constant or some other very small constant and, therefore, in contrast to GR, it describes the very strong correlation between the scalar field ϕ (inflaton field) and χ (geometrical object). So, in the self-consistent problem, small local space-time fluctuations $\delta\phi(x)$ of the ϕ field generate fluctuations $\delta\chi(x)$ of the χ field which, in general, should not be negligible; that is, condition

$$\frac{\delta\chi}{\chi}\ll 1 \tag{32}$$

is not always true. In such a case, the χ field could not be regarded as the background object and, therefore, it is impossible to determine the TMT gravitational background approximation.

One can single out a broad class of situations when the TMT gravitational background approximation has no sense. In the linear approximation in small fluctuations $\delta\phi$, the constraint, Eq. (10), results in

$$\frac{\delta\chi}{\chi}=f(\phi)\,\delta\phi,\quad \text{where}\quad f(\phi)\equiv\frac{V_2'}{V_2}-\frac{V_1'}{V_1+sM^4}. \tag{33}$$

Recall that for the first-class scenarios, the true vacuum state with a zero effective cosmological constant is realized at $\phi=\phi_0$, where $V_1(\phi_0)+sM^4=0$. Then it follows from Eq. (33) that small fluctuations of ϕ in the neighborhood of such vacuum states produces very strong fluctuations of χ. This means that the conception of the TMT gravitational background has no sense in the context of the first class cosmological scenarios and, therefore, the problem of the scalar field ϕ quantization remains without answer.

It is not the case for the second-class cosmological scenarios where $V_1(\phi)+sM^4$ does not equal to zero at any finite value of ϕ. The models of Sec. IV B correspond just to the second-class cosmological scenarios. It is remarkable that the condition, Eq. (32), is satisfied for models of Sec. IV B with extremely high accuracy for all values of ϕ:

[3]On this stage, I restrict myself to models where the matter fields do not contribute to the connection (see Sec. VI C).

$$\text{for model 1,}\quad 0<f(\phi)<\frac{2\beta-\alpha}{M_p}=\frac{8}{M_p};$$

$$\text{for model 2,}\quad \frac{8}{M_p}=\frac{2(\beta-3)}{M_p}<f(\phi)<\frac{2(\beta+1)}{M_p}=\frac{16}{M_p};$$

$$\text{for model 3,}\quad 0<f(\phi)<\frac{15.1}{M_p}.$$

These numerical results mean that the influence of small ϕ fluctuations on χ fields has a typical scale $\delta\phi/M_p$. Thus we conclude that for the models of Sec. IV B, the TMT gravitational background is the well-defined object. Notice that in the variables of the Einstein frame, the TMT gravitational background is described by external fields $g'_{\mu\nu}$ and χ.

VI. INCLUSION OF USUAL MATTER FIELDS

A. Outline of the approach to the problem

Inclusion of the ordinary matter fields (like vector bosons, fermions, etc.) in TMT is a very nontrivial problem. In the framework of the first class cosmological scenarios, it was shown in Ref. [24] that the field theory model exists where, in the conformal Einstein frame, the classical equations of motion of the gauge unified theories as well as the GR equations are exactly reproduced. The merit of this model is that the spontaneous symmetry breaking (SSB) does not generate the cosmological constant term. However, a serious defect of this model consists in the necessity to use the artificial form of how the gauge field kinetic terms and fermion self-interactions enter into the original action. This creates a situation where it is absolutely unclear as to how one can approach the matter fields quantization.

The origin of the problem is practically reduced to the role of the constraint, Eq. (10), which is modified in the presence of usual matter fields. In fact, matter fields in general contribute to the constraint, and then the χ field becomes dependent upon the matter fields. Therefore, when starting with Lagrangians L_1 and L_2, including the matter fields in a form similar to the canonical one, the resulting matter fields equation of motion in the Einstein picture (obtained with the use of the conformal transformations, Eq. (12), or their generalization in the presence of fermions) can appear, in general, to be very nonlinear.

Inclusion of the usual matter fields in the context of the models of Sec. IV B permits us to avoid this problem. In fact, following the idea that the only mass scale typical for the inflaton physics in the limit where the symmetry (18),(19) is exact is the Planck mass, and terms that explicitly break this symmetry contain mass parameters only a few orders of magnitude less than M_p, we provide a situation where *the usual matter field contributions to the constraint appear to be negligible in comparison with the inflaton contributions throughout the history of the universe*. At the late universe, the unbounded increase of the prepotentials (as $\phi\to\infty$) reinforces this effect. As a result of this, the scalar field χ with high accuracy is determined by the same constraint, Eq. (10), as it was in the absence of the usual matter fields. This allows one, starting from the Lagrangians similar to the usual ones, to keep the desirable basic features of the usual matter fields sector after the transition to the Einstein frame. Together with the basic idea about the broken continuous global symmetry, Eqs. (18),(19), modified to the case of the presence of fermions, this approach provides possibilities for constructing gauge models in the context of TMT and, at the same time, to solve problems 1–5 of the Introduction.

B. Action of a gauge Abelian model and continuous global symmetry

In the framework of the formulated above general ideas, let us consider a toy model that possesses gauge Abelian symmetry and contains the following matter fields: a complex scalar field $\xi=1/\sqrt{2}(\xi_1+i\xi_2)$, an Abelian gauge vector field A_μ, and a fermion Ψ. Generalization to non-Abelian gauge theories can be performed straightforward.

In the presence of fermions, the vierbein-spin-connection formalism [32,33] has to be used instead of the first order formalism of Sec. II. The action of the model has the general form as in Eq. (3), with

$$\begin{aligned} L_1=&-\frac{1}{\kappa}R(\omega,V)+\frac{1}{2}g^{\mu\nu}\phi_{,\mu}\phi_{,\nu}-V_1(\phi,|\xi|)\\ &+g^{\mu\nu}(\partial_\mu-ieA_\mu)\xi(\partial_\nu+ieA_\nu)\xi^*\\ &+\frac{i}{2}\overline{\Psi}\left\{\gamma^a V_a^\mu\left(\vec{\partial}_\mu+\frac{1}{2}\omega_\mu^{cd}\sigma_{cd}-ieA_\mu\right)\right.\\ &\left.-\left(\overleftarrow{\partial}_\mu-\frac{1}{2}\omega_\mu^{cd}\sigma_{cd}+ieA_\mu\right)\gamma^a V_a^\mu\right\}\Psi, \end{aligned} \tag{34}$$

$$L_2=V_2(\phi)-\tfrac{1}{4}g^{\alpha\beta}g^{\mu\nu}F_{\alpha\mu}F_{\beta\nu}-h\overline{\Psi}\Psi|\xi|e^{\gamma\phi/M_p}. \tag{35}$$

Here, the following definitions are used [32]:

$$R(\omega,V)=V^{a\mu}V^{b\nu}R_{\mu\nu ab}(\omega),$$

$$R_{\mu\nu ab}(\omega)=\partial_\mu\omega_{\nu ab}+\omega^c_{\mu a}\omega_{\nu cb}-(\mu\leftrightarrow\nu), \tag{36}$$

where $V^{a\mu}=\eta^{ab}V_b^\mu$, η^{ab} is the diagonal 4×4 matrix with elements $(1,-1,-1,-1)$ on the diagonal, V_μ^a are the vierbeins, and $\omega_\mu^{ab}=-\omega_\mu^{ba}$ $(a,b=0,1,2,3)$ is the spin connection.

Prepotential $V_2(\phi)$ is the same as in the models of Sec. IV B. Prepotential $V_1(\phi,|\xi|)$ is chosen in the form

$$V_1(\phi,|\xi|)=V_1(\phi)+P(|\xi|)e^{\alpha\phi/M_p}, \tag{37}$$

where $V_1(\phi)$ is the same as in the models of Sec. IV B; that is, $e^{\alpha\phi/M_p}$ is the common factor in front of $m_1^4+P(|\xi|)$ in Eq. (37).

The transformations of the continuous global symmetry, Eqs. (18),(19), are generalized now to the form [26]

$$\begin{aligned} &V_a^\mu\to e^{-\theta/2}V_a^\mu,\quad g_{\mu\nu}\to e^\theta g_{\mu\nu},\quad A_{\mu\nu\lambda}\to e^\theta A_{\mu\nu\lambda},\\ &\phi\to\phi-\frac{M_p}{\beta}\theta,\quad \xi\to\xi,\quad A_\mu\to A_\mu,\\ &\Psi\to e^{-\theta/4}\Psi,\quad \overline{\Psi}\to e^{-\theta/4}\overline{\Psi}. \end{aligned} \tag{38}$$

The term $\int P(|\xi|)e^{\alpha\phi/M_p}\Phi d^4x$ breaks the symmetry (38) by the same manner as the prepotential $V_1(\phi)$. For the Yukawa coupling type term

$$S_{Yuk}=-h\int\overline{\Psi}\Psi|\xi|e^{\gamma\phi/M_p}\sqrt{-g}d^4x \tag{39}$$

to be invariant under transformations (38), the parameter γ must be $\gamma=\frac{3}{2}\beta$. The value of γ preferable from the dynamical point of view will be discussed later, and we will see that $\gamma<2\beta$. All other terms describing the usual matter fields are invariant under transformations (38). If $\gamma\neq\frac{3}{2}\beta$, then the symmetry is explicitly broken only by the Yukawa coupling type term and the prepotentials $V_1(\phi,|\xi|)$ and $V_2(\phi)$. Thus, similar to the models of Sec. IV B, in the model with the Lagrangian densities (34) and (35), the global continuous symmetry (38) is restored as $\phi\to\infty$.

It is interesting that the form of the ϕ dependence of the Yukawa type term dictated by the symmetry (38) is very similar to a motivated by string theories nucleon-scalar coupling discussed by Wetterich [6] in the context of a quintessence type model with exponential potential.

Note finally that for pedagogical reasons we have started from the simplified model where the Yukawa type term appears only with the measure $\sqrt{-g}$. We will see later (see Sec. VI H) that an additional Yukawa type term in Eq. (34), that is, with the measure Φ, is needed to provide the possibility to avoid the long-range force problem.

C. Connection, equations of motion, and constraint

Variation of the action with respect to ω_μ^{ab} leads to the equation a solution of which is represented in the form [24]

$$\omega_\mu^{ab}=\omega_\mu^{ab}(V)+K_\mu^{ab}(\sigma)+K_\mu^{ab}(V,\overline{\Psi},\Psi), \tag{40}$$

where $\omega_\mu^{ab}(V)$ is the Riemannian part of the connection [32,33], and

$$K_\mu^{ab}(\sigma)=\frac{1}{2}\sigma_{,\alpha}(V_\mu^aV^{b\alpha}-V_\mu^bV^{a\alpha}),\quad \sigma\equiv\ln\chi, \tag{41}$$

$$K_\mu^{ab}(V,\overline{\Psi},\Psi)=\frac{\kappa}{8}\eta_{ci}V_{d\mu}\varepsilon^{abcd}\overline{\Psi}\gamma^5\gamma^i\Psi. \tag{42}$$

For brevity, we omit here equations obtained by variations of vierbeins, $A_{\mu\nu\lambda}$, as well as of the matter fields ϕ, ξ, A_μ, Ψ, and $\overline{\Psi}$. Combining equations obtained by variation of vierbeins and $A_{\mu\nu\lambda}$, and using equations of motion for Ψ and $\overline{\Psi}$, one can eliminate $R(\omega,V)$, and the result is the constraint

$$sM^4+V_1(\phi)+P(\varphi)e^{\alpha\phi/M_p} = \frac{2}{\chi}\left[V_2(\phi)-\frac{3}{4\sqrt{2}}h\overline{\Psi}\Psi\varphi e^{\gamma\phi/M_p}\right], \qquad (43)$$

which is a direct generalization of the constraint (10) to the model we study here.

One of the aims of this toy model consists in a demonstration of the possibility to construct realistic gauge unified theories (such as electroweak and GUT) in the context of cosmological scenarios dictated by models of Sec. IV B. Introducing the scalar field ξ is intended for the realization of the Higgs phenomenon. Since $P(\varphi)$ and m_1^4 appear in the combination $m_1^4+P(\varphi)$, the constant part of $P(\varphi)$ can be always absorbed by m_1^4. Then it is natural to assume[4] that $|P(\varphi)|\ll m_1^4$. Later, turning to quantum effective potential, we will discuss a concrete model where $P(\varphi)=(\overline{\lambda}/4!)\varphi^4$, and then the idea explained in Sec. VI A will become clearer. The choice of the mass parameters in the models of Sec. IV B allows us to provide a situation where the contribution of the Higgs field φ to the constraint (43) is negligible with respect to the inflaton field ϕ-contribution and, hence, it can give only extremely small corrections to the main picture. If fluctuations of fermionic fields are not abnormally large, it is natural to expect that the same conclusion is true for fermionic contributions to the constraint (43) as well. So, the χ field determined by the constraint (43), in practically interesting cases, coincides with the χ field determined by the constraint (10), which holds in the model free of the usual matter. For brevity, in what follows, when neglecting the usual matter fields contribution to the constraint, we will use the term *A* approximation. This notion will be very useful in the next subsection, where we are going to represent equations of motion in the Einstein frame.

D. Equations of motion for the self-consistent problem in the Einstein frame

In the presence of fermions, the transition to the Einstein frame (a more suitable term for this case would be the Einstein-Cartan frame) is carried out by the transformations to the new variables [24]

$$V_{a\mu}(x)\to V'_{a\mu}(x)=\chi^{1/2}(x)V_{a\mu}(x),$$
$$g_{\mu\nu}(x)\to g'_{\mu\nu}(x)=\chi(x)g_{\mu\nu}(x),$$
$$\Psi(x)\to\Psi'(x)=\chi^{-1/4}(x)\Psi(x),$$
$$\overline{\Psi}(x)\to\overline{\Psi}'(x)=\chi^{-1/4}(x)\overline{\Psi}(x),$$
$$\phi\to\phi, \quad A_{\mu\nu\lambda}\to A_{\mu\nu\lambda},$$
$$\varphi\to\varphi, \quad A_\mu\to A_\mu, \qquad (44)$$

where χ is determined by the constraint (43).

In fact, after the transition to the new variables defined by the transformations (44), the σ contribution, Eq. (41), to the spin connection is canceled, and the transformed spin connection takes the form [24]

$$\omega_\mu^{\prime cd}=\omega_\mu^{cd}(V')+\frac{\kappa}{8}\eta_{ci}V'_{d\mu}\varepsilon^{abcd}\overline{\Psi}'\gamma^5\gamma^i\Psi', \qquad (45)$$

which coincides with the well-known solution for the spin connection in the context of the first order formalism approach to the Einstein-Cartan theory [32], where a Dirac spinor field is the only source of a non-riemannian part of the connection. Hence, the curvature tensor, Eq. (36), expressed in terms of the new connection, Eq. (45), becomes the curvature tensor of such an Einstein-Cartan theory.[5]

At the same time, in the fermionic field equation, all terms containing $\sigma_{,\mu}$ also disappear [24] in the Einstein-Cartan frame, and the result is

$$\left\{i\left[V_a^{\prime\mu}\gamma^a(\partial_\mu-ieA_\mu)+\gamma^aC_{ab}^{\prime b}+\frac{i}{4}\omega_\mu^{\prime cd}\varepsilon_{abcd}\gamma^5\gamma^bV^{\prime a\mu}\right] -\frac{h}{\sqrt{2}}\varphi\frac{e^{\gamma\phi/M_p}}{\chi^{3/2}}\right\}\Psi'=0, \qquad (46)$$

where $C_{ab}^{\prime b}$ is the trace of the Ricci rotation coefficients [32] in the new variables, and the unitary gauge is used. After a shift, we define

$$\xi=\frac{1}{\sqrt{2}}\varphi\equiv\frac{1}{\sqrt{2}}(\upsilon+\tilde{\varphi}(x)), \quad \upsilon=\text{const.} \qquad (47)$$

The equation for $\overline{\Psi}'$ has a similar structure. The only difference between these fermionic equations and the standard Dirac equations in the Einstein-Cartan theory [32] is related to an unusual Yukawa type term, and it will be discussed

[4]Recall that m_1 appears in the definition of the prepotential $V_1(\phi)$ in the models of Sec. IV B. The values of m_1 are chosen such that $m_1^4=(10^{-2}M_p)^4$ in model one, and $m_1^4=(10^{-3}M_p)^4$ in models two and three.

[5]Notice that in the original frame, the terms including $\sigma_{,\mu}$ (recall that $\sigma\equiv\ln\chi$) originate a nonmetricity and, therefore, TMT in the original variables has no form of an Einstein-Cartan theory.

later. Notice that for purposes of realistic particle physics one can neglect the second term in Eq. (45) that leads to a spin-spin contact interaction [32] with coupling constant M_p^{-2}. For brevity, in what follows, when neglecting this interaction, we will use the term B approximation.

Other equations of motion in the Einstein-Cartan frame have the following form:

$$\frac{1}{\sqrt{-g'}}\partial_\mu(\sqrt{-g'}g'^{\mu\nu}\partial_\nu\phi)+\frac{1}{\chi}\left[\frac{dV_1}{d\phi}-\frac{1}{\chi}\frac{dV_2}{d\phi}+\frac{\alpha}{M_p}P(\varphi)e^{\alpha\phi/M_p}\right]=-\frac{h\gamma}{\sqrt{2}M_p}\overline{\Psi}'\Psi'\varphi\frac{e^{\gamma\phi/M_p}}{\chi^{3/2}}, \tag{48}$$

$$\frac{1}{\sqrt{-g'}}\partial_\mu(\sqrt{-g'}g'^{\mu\nu}\partial_\nu\tilde{\varphi})+\frac{e^{\alpha\phi/M_p}}{\chi}\frac{dP(\varphi)}{d\varphi}-e^2\varphi g'^{\alpha\beta}A_\alpha A_\beta=-\frac{h}{\sqrt{2}}\overline{\Psi}'\Psi'\frac{e^{\gamma\phi/M_p}}{\chi^{3/2}}, \tag{49}$$

$$\frac{1}{\sqrt{-g'}}\partial_\mu(\sqrt{-g'}g'^{\mu\alpha}g'^{\nu\beta}F_{\alpha\beta})+\frac{e^2}{2}\varphi^2 g'^{\mu\nu}A_\mu=-e\overline{\Psi}'\gamma^a V_a'^{\nu}\Psi'. \tag{50}$$

It is very important to stress that in the A and B approximations, all matter fields equations, (46),(48)–(50), have the canonical structure of the corresponding matter fields equations in a Riemannian space-time. The only specific features of these equations are concentrated in the unusual forms of the effective potentials and some of the interactions.

After some algebraic manipulations with equations resulting from variation of the vierbeins, transition to the new variables by means of Eq. (44), and making use of both the fermionic equation (46) and a similar equation for $\overline{\Psi}'$, we obtain canonical gravitational equations of the Einstein-Cartan theory. Finally, if one writes down these equations in the B approximation, we come to the canonical GR gravitational equation

$$G_{\mu\nu}=\frac{\kappa}{2}T_{\mu\nu}, \tag{51}$$

where $G_{\mu\nu}$ is the Einstein tensor of the Riemannian space-time with metric $g'_{\mu\nu}$, and the energy-momentum tensor has a canonical GR structure [27]

$$\begin{aligned}T_{\mu\nu}=&\phi_{,\mu}\phi_{,\nu}-\frac{1}{2}g'_{\mu\nu}\phi_{,\alpha}\phi_{,\beta}g'^{\alpha\beta}+\frac{1}{\chi^2}V_2(\phi)g'_{\mu\nu}\\&+\tilde{\varphi}_{,\mu}\tilde{\varphi}_{,\nu}-\frac{1}{2}g'_{\mu\nu}\tilde{\varphi}_{,\alpha}\tilde{\varphi}_{,\beta}g'^{\alpha\beta}\\&+\frac{1}{4}g'_{\mu\nu}F_{\alpha\beta}F_{\tau\rho}g'^{\alpha\tau}g'^{\beta\rho}-F_{\mu\alpha}F_{\nu\beta}g'^{\alpha\beta}\\&+e^2(v+\tilde{\varphi})^2\left(A_\mu A_\nu-\frac{1}{2}g'_{\mu\nu}A_\alpha A_\beta g'^{\alpha\beta}\right)\\&+\frac{i}{2}[\overline{\Psi}'\gamma^a V'_{a(\mu}\nabla_{\nu)}\Psi'-(\nabla_{(\mu}\overline{\Psi}')\gamma^a V'_{\nu)a}\Psi'],\end{aligned} \tag{52}$$

where $\nabla_\mu\Psi'=(\partial_\mu+\frac{1}{2}\omega_\mu'^{cd}\sigma_{cd}-ieA_\mu)\Psi'$ and $\nabla_\mu\overline{\Psi}'=\partial_\mu\overline{\Psi}'-\frac{1}{2}\omega_\mu'^{cd}\overline{\Psi}'\sigma_{cd}+ieA_\mu\overline{\Psi}'$.

Notice again that the χ field entering into Eqs. (46),(48),(49), and (52) is determined by the constraint (43) which in the A approximation gives

$$\frac{1}{\chi}=\frac{M^4+V_1(\phi)}{2V_2(\phi)}. \tag{53}$$

In what follows, all discussions will be performed in the framework of the A and B approximations.

It is worthwhile to notice that the transformations of the global continuous symmetry, Eq. (38), expressed in terms of the variables of the Einstein frame, are just reduced to shifts of ϕ: $\phi\to\phi-(M_p/\beta)\theta$.

E. Effective classical action for usual matter fields in the background

To study the matter fields sector of the system of Eqs. (46)–(53), one has to define an appropriate background. In Sec. V we discused two different gravitational backgrounds in the model where the usual matter was absent and the inflaton field ϕ was the only field of the nongravitational sector. One can see that if we proceed with the formal gravitational background, then it will be impossible to write down an effective classical action in the curved background giving rise to the system of Eqs. (46),(48)–(50). For example, to provide the appearance of the last term of Eq. (46) and the right-hand sides of Eqs. (48) and (49), such an effective classical action in the curved background has to include the Yukawa coupling type term

$$L_{Yuk}=-\frac{h}{\sqrt{2}}\overline{\Psi}'\Psi'\varphi\frac{e^{\gamma\phi/M_p}}{\chi^{3/2}}.$$

Working in the formal gravitational background, we have to insert the expression for χ, Eq. (53), into L_{Yuk}. But then variation of the inflaton field ϕ leads not only to the appearance of the needed terms, but the unwanted terms, coming from the variation of $\chi(\phi)$, will appear as well. It is not the case in the framework of the TMT gravitational background since in that case the scalar field χ is the background one.

If, however, we want to construct the quantum theory of the usual matter fields, then it seems to be natural to start from the approximation where in addition to the gravitational background, the inflaton field ϕ is also regarded as the background one. This can be done since in the course of its evolution, the classical inflaton field ϕ remains practically constant during a typical time of quantum fluctuations of the matter fields. In such a case, the above mentioned difference between two definitions of the gravitational background disappears: the background field χ is determined by the background field ϕ via the constraint, Eq. (53).

So, let us study some features of the particle physics model in the background that, in terms of variables of the Einstein picture, consists of two external fields: $g'_{\mu\nu}$ and ϕ. For brevity, I will refer to this issue as the particle physics model in the cosmological background.

The effective classical action for the particle physics model corresponding to the system of Eqs. (46),(49), and (50), in the cosmological background, can be written down in the following form (in the unitary gauge):

$$S_{class}^{background}=\int\sqrt{-g'}\bigg[\frac{1}{2}g'^{\mu\nu}\varphi_{,\mu}\varphi_{,\nu}-V_{cl}(\varphi;\phi)+\frac{e^2}{2}\varphi^2A_\mu A_\nu g'^{\mu\nu}-\frac{1}{4}g'^{\alpha\beta}g'^{\mu\nu}F_{\alpha\mu}F_{\beta\nu}+L_{kin}(\overline{\Psi}',\Psi',A_\mu)+L_{Yuk}(\overline{\Psi}'\Psi'\varphi;\phi)\bigg], \tag{54}$$

where $V_{cl}(\varphi;\phi)$ is the classical TMT effective potential for the matter (Higgs) scalar field φ in the presence of the background inflaton field ϕ,

$$V_{cl}(\phi;\varphi)=P(\varphi)\frac{M^4+V_1(\phi)}{2V_2(\phi)}e^{\alpha\phi/M_p}, \tag{55}$$

$L_{kin}(\overline{\Psi}',\Psi',A_\mu)$ is the standard kinetic term for the fermion field in a Riemannian space-time with metric $g'_{\mu\nu}$, also including the gauge coupling to the vector field A_μ. And finally, the TMT effective Yukawa coupling type term $L_{Yuk}(\overline{\Psi}'\Psi'\varphi;\phi)$ is

$$L_{Yuk}(\overline{\Psi}'\Psi'\varphi;\phi)=-\frac{h}{\sqrt{2}}\overline{\Psi}'\Psi'\varphi\frac{e^{\gamma\phi/M_p}}{\chi^{3/2}}=-\frac{h}{4}\overline{\Psi}'\Psi'\varphi\left[\frac{M^4+V_1(\phi)}{V_2(\phi)}\right]^{3/2}e^{\gamma\phi/M_p}. \tag{56}$$

F. Massless scalar electrodynamics model in the cosmological background and SSB

Up to now the function $P(\varphi)$ was unspecified. Ignoring the technical questions (in particular, the question of renormalizability that requires a nonminimal coupling $\eta R|\varphi|^2$), let us attract attention to a quantum effective potential when choosing $P(\varphi)=(\overline{\lambda}_0/4!)\varphi^4$, $\overline{\lambda}_0=\text{const}$. This means that (ignoring the fermion field) we are dealing with massless scalar electrodynamics in curved space-time, where the classical potential (the tree approximation) is given by

$$V_{cl}(\varphi;\phi)=\frac{\lambda_0(\phi)}{4!}\varphi^4 \tag{57}$$

and $\lambda_0(\phi)$ depends on the background field ϕ:

$$\lambda_0(\phi)=\overline{\lambda}_0k(\phi),\quad k(\phi)\equiv\frac{M^4+V_1(\phi)}{2V_2(\phi)}e^{\alpha\phi/M_p}. \tag{58}$$

Numerical estimations of $k(\phi)$ in the models of Sec. IV B give the following results: $0<k(\phi)<3.5$ for model one; $0<k(\phi)<1.2\times10^{-8}$ for model two; and $0<k(\phi)<3\times10^{-7}$ for model three. In all models, $k(\phi)$ asymptotically approaches zero as $\phi\to\pm\infty$. Thus, in all cases, $\lambda_0(\phi)$ is of the same order or less than $\overline{\lambda}_0$.

The computation technics of the effective potential for the massless scalar electrodynamics in the one-loop approximation is a well-known issue [34]. However, the problem we study here is not quite usual: the quartic coupling constant depends actually on the cosmic time via the inflaton field ϕ. Taking into account that in the course of its evolution, the classical field ϕ remains practically constant during a typical time of quantum matter fields fluctuations, it is natural to consider the problem in the adiabatic approximation. Therefore, computing the effective potential we can regard $\lambda_0(\phi)$ as a constant. Then the computation becomes quite standard. The only additional issue we have to clear up is a possible physical effect that the adiabatically changing $\lambda_0(\phi)$ might be on the φ-effective potential.

One can check that the first point where we encounter necessity to decide this problem is the renormalization procedure. In fact, performing calculations with the bare coupling constant λ_0, we have no need to think about its adiabatic ϕ dependence. But when we turn to the use of the renormalized (finite) parameter λ defined by $\lambda_0=\lambda+\delta\lambda$, where $\delta\lambda$ is the counterterm (which, as one knows, is divergent in perturbation theory), we have to take into account a possible ϕ dependence of the effective λ.

The vector boson loops contribution to the effective potential in the one-loop approximation has the order of e^4 and does not depend on ϕ [see Eq. (54)]. Therefore, just as in the standard scalar electrodynamics, one can assert that in spite of the possibility for $\lambda_0(\phi)$ to be very small, the effective $\lambda(\phi)$ cannot be too small. On the other hand, it is also important that $\lambda(\phi)$ cannot be large: since ϕ dependence of λ_0 acts in the decreasing direction in comparison with $\overline{\lambda}_0$, there are no reasons for a possible ϕ dependence of λ to act in the opposite direction.

The scalar loops contribution has the order of λ^2. Therefore, in the same way as in the standard scalar electrodynamics, in the one-loop approximation, one can neglect the scalar loops contribution with respect to the vector boson loops contribution.

The one-loop effective potential for the scalar field φ evaluated at the fixed value of the background inflaton field $\phi=\phi_1$ can be written in the form

$$V_{eff}(\varphi;\phi_1)=\frac{\lambda(\phi_1)}{4!}\varphi^4+\frac{3e^4}{(8\pi)^2}\varphi^4\left(\ln\frac{\varphi^2}{\mu^2}-\frac{25}{6}\right), \tag{59}$$

where

$$\lambda(\phi_1)=\left.\frac{d^4V_{eff}}{d\varphi^4}\right|_{\varphi=\mu}. \tag{60}$$

Let us assume that ϕ_1 is the value of the background inflaton field where $\lambda(\phi)$ has a maximal possible magnitude (but it is still small). Suppose also that the renormalization mass μ is chosen such that $\lambda(\phi_1)\sim e^4$. This can always be done, as is well known from the renormalization group analysis [34]. The final form of the effective potential

$$V_{eff}(\varphi;\phi_1)=\frac{3e^4}{(8\pi)^2}\varphi^4\left(\ln\frac{\varphi^2}{v^2}-\frac{1}{2}\right) \tag{61}$$

is determined in terms of two free parameters: the renormalized gauge coupling constant e and VEV $\langle\varphi\rangle=v$.

To verify whether the change of the value of the background inflaton field ϕ has some physical consequences, let us suppose that we want to repeat the same computation of the one-loop effective potential at another fixed value of the background inflaton field $\phi=\phi_2$, where the order of the magnitude of $\lambda(\phi_2)$ is less than e^4 if we take the same renormalization mass μ. According to the results of the renormalization group analysis [34], one can move $\lambda(\phi_2)$ to the magnitude of the order of e^4 by a change in the renormalization mass that does not change the order of magnitude of e. This can always be done if $\lambda(\phi_2)$ is small. Then the computation of the one-loop effective potential at $\phi=\phi_2$ in the same approximation leads to the same effective potential as it was at $\phi=\phi_1$, Eq. (61), with the same order of magnitude of the free parameter e. One can conclude, therefore, that in the used approximation, the ϕ dependence of λ has no physical effect.

In this stage of the investigation, I will ignore the fermion loop contribution into the ϕ effective potential. The nonminimal coupling of the Higgs field φ to curvature, which appears in the quantum effective action in curved space-time [35], might have some interesting but, most likely, weak enough effect, and this question exceeds the limits of the present paper.

So, for the usual form of the function $P(\varphi)$, we obtain, in a cosmological background, the effective quantum potential for the scalar (Higgs) field φ typical for gauge theories with dynamical symmetry breaking. Notice again that the term $(e^2/2)\varphi^2A_\mu A_\nu g'^{\mu\nu}$ in Eq. (54) does not depend on the inflaton field ϕ. Thus, the SSB and Higgs phenomenon occur in a standard way.

G. Yukawa coupling type term and fermion mass

As a result of SSB, the Yukawa coupling type term, Eq. (56) [see also Eq. (46)], produces the TMT effective fermion mass m_f depending on the inflaton field ϕ:

$$m_f=m_f(\phi)=\frac{h}{4}v\left[\frac{M^4+V_1(\phi)}{V_2(\phi)}\right]^{3/2}e^{\gamma\phi/M_p}. \tag{62}$$

For $\phi>M_p$ (the region corresponding to the late universe), the fermion mass becomes

$$m_f^{(late)}\simeq m_f^{(0)}e^{-[3(\beta-\frac{1}{2}\alpha)-\gamma]\phi/M_p},$$

$$m_f^{(0)}=2hv\left(\frac{m_1}{m_2}\right)^6,\quad \text{as}\quad \phi>M_p. \tag{63}$$

We see that in the late universe, the fermion mass approaches the nonzero constant $m_f^{(0)}$ if

$$\gamma=3\left(\beta-\frac{\alpha}{2}\right). \tag{64}$$

Notice that if γ indeed satisfies the relation (64), then with the choice as in Sec. IV (i.e., $\alpha=6$ and $\beta=7$), we obtain $\gamma=12$, which is close to the value of $\frac{3}{2}\beta=10.5$ dictated by the symmetry (38) [see the discussion after Eq. (39)].[6] So, in the framework of our working hypothesis about approximate symmetry (38), one can ensure *a successful mass generation for fermions in the present cosmological epoch in a way typical for the standard model* and, at the same time, one can keep the direct coupling, Eq. (56), of fermionic matter to the inflaton field (compare this with Wetterich's model [6]).

One has to notice that a formal generalization of the toy (Abelian) model we study here, to a non-Abelian model [like $SU(2)\times U(1)$ or $SU(5)$] can be performed straightforward. Then we have to worry about scales of the particle mass generated as a result of SSB. In this connection it would be interesting to estimate the order of magnitude of the fermion mass in the present universe that one could expect on the basis of Eqs. (63) and (64). With the mass parameters m_1 and m_2 of the models of Sec. IV B (that implies $m_1/m_2=10^{-2}$ for model one and $m_1/m_2=10^{-3}$ for models two and three), and with $v\sim10^2$ GeV, estimates give too small values for fermion mass at the late universe: $m_f^{(0)}\sim h\times10^{-1}$ eV in model one and $m_f^{(0)}\sim h\times10^{-7}$ eV in models two and three. This is because of the presence, in Eq. (63), of the very small factor $(m_1/m_2)^6$.

Masses of the vector bosons, as it was explained at the end of the previous subsection, do not depend on the inflaton

[6]The value $\gamma=12$ is as close to $\frac{3}{2}\beta=10.5$ as $\beta=7$ is close to $\alpha=6$.

field ϕ, and their values are defined as in the standard gauge unified models. For the mass generation of fermions we have more freedom than in the standard models. According to the basic ideas of the model developed in the present paper, the general structure of Eq. (62) for masses of fermions is the same for field theory models with different symmetry groups. The only free parameters, besides the inflaton field ϕ, are the VEV v of the appropriate scalar boson and γ. If the values of the $\gamma's$ are determined by Eq. (64), then masses of all fermions in the present universe are constants. If, however, the parameter γ corresponding to some of the fermions is such that $3(\beta-\frac{1}{2}\alpha)-\gamma$ is very small but nonzero, then m_f becomes slow ϕ dependent according to Eq. (63) even at the late universe. Namely, since in the quintessence model with exponential potential, Eq. (29), the inflaton field ϕ changes [6] in cosmic time as $\phi \propto (M_p/(\beta-\alpha))\ln t$, we obtain that $m_f(\phi(t))$ will change in such a case as $t^{-[3(\beta-1/2\alpha)-\gamma]/(\beta-\alpha)}$. If $|3(\beta-\frac{1}{2}\alpha)-\gamma| \ll \beta-\alpha$ (in the models of Sec. IV B, this means $|12-\gamma| \ll 1$), the rate of change of m_f might be very small in the present universe. Depending on the sign of $3(\beta-\frac{1}{2}\alpha)-\gamma$, which should not be the same for all fermions, m_f could be either increasing or decreasing. Notice that the case $3(\beta-\frac{1}{2}\alpha)-\gamma \leq 0$ corresponds in some sense to the model studied by Wetterich [6].

Concerning the very early universe, that is, for $\phi < -M_p$, one can see that the model predicts the TMT effective fermion mass, Eq. (62), to be extremely small: $m_f \to 0$ as $\phi \to -\infty$. For example, in model three of Sec. IV B, $m_f \simeq h v 10^{-2} e^{-\gamma|\phi|/M_p}$ as $\phi < -M_p$. At the same time, the gauge coupling of Ψ' to A_μ [see Eq. (46)] is the standard one and, in particular, it does not depend on the inflaton field ϕ.[7]

H. The long-range force problem

The right-hand side of Eq. (48) describes a model with direct coupling of the inflaton to fermionic matter. For all models of Sec. IV B at the present universe, i.e., in the quintessential region (see Sec. IV C, item three), the effective Lagrangian of this coupling takes the form

$$L^{(Yuk)}_{eff,present} = -\gamma \frac{m_f^{(late)}}{M_p} \overline{\Psi}' \Psi' \phi. \tag{65}$$

Assuming the condition (64) for constancy of $m_f^{(late)}$, and with the choice $\beta=7$, $\alpha=6$, we get that the coupling constant of the present day effective Yukawa coupling of inflaton to fermion is $12 m_f^{(0)}/M_p$. Existence of such a coupling would produce too strong a scalar long-range force. Fortunately, TMT gives us additional tools that allow to solve this problem.

In the model [26] with only spontaneous breaking of the global continuous symmetry (38), Guendelman studied the case where the direct fermion-inflaton couplings similar to Eq. (39) are present in the original TMT action, both with the measure Φ and with the measure $\sqrt{-g}$. In such a model the constant fermion mass is also achieved [26]. Having this idea in mind, let us modify our model, Eqs. (34),(35), with the explicit breaking of the symmetry (38), by including an additional Yukawa coupling type term which enters into the action with the measure Φ,

$$\tilde{S}_{Yuk} = -\tilde{h} \int \overline{\Psi}\Psi |\xi| e^{\tilde{\gamma}\phi/M_p} \Phi d^4x. \tag{66}$$

For this term to be invariant under transformations (38), the parameter $\tilde{\gamma}$ must be $\tilde{\gamma} = \frac{1}{2}\beta < \alpha$. The magnitude of $\tilde{\gamma}$ preferable from the dynamical point of view will be discussed below.

One can check that in this modified model, the fermion mass at the late universe becomes

$$m^{(late)}_{f,modified} \simeq v \left(\frac{m_1}{m_2}\right)^2 \left[2h \left(\frac{m_1}{m_2}\right)^4 e^{-[3(\beta-\frac{1}{2}\alpha)-\gamma]\phi/M_p} + \tilde{h} e^{-(\beta-\frac{1}{2}\alpha-\tilde{\gamma})\phi/M_p} \right] \quad \text{as} \quad \phi > M_p. \tag{67}$$

The constancy of $m^{(late)}_{f,modified}$ is achieved now if the condition

$$\tilde{\gamma} = \beta - \frac{1}{2}\alpha \tag{68}$$

holds together with Eq. (64). For $\beta=7$ and $\alpha=6$, the constancy of the fermion mass at the late universe implies that $\tilde{\gamma}=4$, which is as close to $\tilde{\gamma}=\frac{1}{2}\beta=3.5$ as α is close to β.

With the conditions for constancy of the fermion mass at the late universe, Eqs. (64) and (68), the modified effective Yukawa coupling of the inflaton to fermionic matter now takes the form

$$L^{(Yuk,modified)}_{eff,present} = -\frac{v}{M_p}\left(\frac{m_1}{m_2}\right)^2 \left(\beta - \frac{1}{2}\alpha\right) \times \left[6h \left(\frac{m_1}{m_2}\right)^4 + \tilde{h} \right] \overline{\Psi}' \Psi' \phi. \tag{69}$$

We see that in the modified model there exists a possibility to prevent the appearance of such dangerous interaction. To realize this opportunity we have to require

$$\frac{\tilde{h}}{h} = -6 \left(\frac{m_1}{m_2}\right)^4. \tag{70}$$

This is actually strong enough tuning since, for instance, in the context of models two and three of Sec. IV B, it implies $|\tilde{h}/h| \sim 10^{-12}$. If we recall that $\tilde{h}$ and h are the Yukawa type coupling constants of the Higgs scalar to the fermion, it appears surprising that their ratio has to be of the order of magnitude that shows the degree of the hierarchy problem in GUT: $m_W/m_X \sim 10^{-12}$.

With conditions (64),(68), and (70), the fermion mass at the late universe becomes

[7] This can be an interesting example of the model [36] of massless spinor electrodynamics realized as the limit of a massive theory as $\phi \to -\infty$.

$$m_{f,modified}^{(late)}=\frac{2}{3}\tilde{h}v\left(\frac{m_1}{m_2}\right)^2 \quad \text{as} \quad \phi > M_p. \tag{71}$$

A possible relation of the discussed question to the hierarchy problem in GUT, as well as other problems that appear in the attempts to generate a realistic unified gauge theory in the context of TMT, will be studied elsewhere.

VII. DISCUSSION AND CONCLUSION

Before summarizing and discussing the main results of this paper, I would like to stress again that the first impression that the studied models belong to a sort of scalar-tensor theory, is wrong. The ratio of two measures, that is, the scalar field χ, Eq. (9), is the only object entering into the equations of motion and carrying information about the measure Φ degrees of freedom. If we restrict ourselves to models where L_1 is linear in the scalar curvature [see Eqs. (3),(5), and (34)] and L_2 does not contain curvature, then in the first order formalism, a constraint appears which determines χ in terms of matter fields [see Eq. (10) or (43)]. This means that in such models, the scalar field χ does not carry an independent degree of freedom. All deviations from the Einstein or Einstein-Cartan theory existing in the original variables are caused by derivatives of $\sigma \equiv \ln \chi$, and they disappear in the new variables obtained by the conformal transformations, Eq. (12) or (44). By an appropriate choice of L_1 and L_2 one can provide that all equations of motion in the new variables have canonical GR forms of equations for gravity and matter fields. All novelty is revealed only in an unusual structure of the effective potentials and interactions. And just this novelty enables us to solve a number of problems (questions 1–5 of the Introduction), most of which in the framework of GR require fine-tuning.

(a) Towards a resolution of the cosmological constant problem. Let us return, for the moment, to the simple model of Sec. II. If one takes [24] $V_2(\phi)\equiv -\Lambda = \text{const}$, which would correspond[8] to a model with a cosmological constant Λ in GR then we see that the greater $|\Lambda|$ we admit, the smaller the TMT effective potential, Eq. (13), we obtain in the Einstein picture. This is a direct result of the existence of two measures and two Lagrangians in the original TMT action, Eq. (5). We see that TMT turns over our intuitive ideas based on our experience in field theory.

The resolution of the cosmological constant problem in models studied in Refs. [23–26] was based on the assumption that a cosmological scenario belongs to the first class (see Sec. II). In the context of such types of scenarios, those TMT models predict that if $V_2(\phi)$ is positive definite, the stable vacuum with zero energy density is realized without any sort of fine tuning at a finite value of $\phi=\phi_0$, where $V_1(\phi_0)+sM^4=0$. As we have seen in Sec. V, in such a vacuum the usual conception of the gravitational background becomes invalid, and small fluctuations of ϕ cause infinitely large fluctuations of χ. For the true vacuum state this feature is unacceptable.

For this reason, in this paper we studied cosmological scenarios of the second class (see Sec. II), where the true vacuum state is realized asymptotically as $\phi \to \infty$. This naturally leads to a need to apply a quintessence model of the late universe. However, in contrast to quintessence models studied in the framework of GR or Brans-Dicke type models, in TMT we have a new option: one can choose the prepotentials V_1 and V_2 as increasing at the late universe (that is, as $\phi > M_p$). If V_1^2/V_2 approaches zero as $\phi\to\infty$, then the TMT effective potential (13) asymptotically approaches zero at the late universe. One can adjust degrees of growth of V_1 and V_2 in such a way that the TMT effective potential $U(\phi)$ will have a desirable flat shape as $\phi\to\infty$. Unbounded growth of V_2 as $\phi\to\infty$ allows adding to V_2 any constant V_2^0 without altering $U(\phi)$ for large enough ϕ (remind that the appearance of an additive constant in V_1 does not affect equations of motion at all). This is actually what we have seen in Sec. III. If the appearance of the appropriate term $\int V_2^0\sqrt{-g}d^4x$ in the action is a result of quantum vacuum fluctuations, then we can conclude that in the framework of the described approach to constructing a quintessence model of the late universe, TMT solves the cosmological constant problem.

However, the impression that the described technical details of the approach to the resolution of the cosmological constant problem in TMT settles a question is premature. One should remind that the last statement about resolution of the cosmological constant problem implies validity of one more basic conjecture formulated in the Introduction [after Eq. (4)], and used in all models of the present paper: Lagrangians L_1 and L_2 in the original action, Eq. (3), do not depend on the measure Φ degrees of freedom. In the cases when this conjecture is invalid, the cosmological constant problem in TMT can turn into a very nontrivial issue. In fact, till the fundamental theory remains unknown, one cannot be sure that the postulated general structure of TMT survives after quantum corrections are taken into account. If it will turn out that the quantum effective action corresponding to the original theory, Eq. (3), contains the term $-\int\Phi\chi\Lambda_{eff}d^4x$, then in the Einstein frame the latter will generate the real cosmological constant Λ_{eff}. This possibility was studied in Ref. [24] (see Sec. VI therein), where a way to prevent the appearance of such a dangerous term was also discussed. The idea, briefly, is the following: If instead of the antisymmetric tensor field $A_{\mu\nu\lambda}$, the measure Φ is defined by means of four scalar measure fields φ_a,($a=1,2,3,4$),

$$\Phi \equiv \varepsilon_{a_1a_2a_3a_4}\varepsilon^{\mu\nu\lambda\sigma}(\partial_\mu\varphi_{a_1})(\partial_\nu\varphi_{a_2})(\partial_\lambda\varphi_{a_3})(\partial_\sigma\varphi_{a_4}), \tag{72}$$

then the action, Eq. (3), with φ_a independent L_1 and L_2, is invariant up to an integral of a total divergence under transformations $\varphi_a\to\varphi_a+f_a(L_1)$ where $f_a(L_1)$ are arbitrary differentiable functions of L_1. An appearance of the danger

[8]Taking into account our definition of $V_2(\phi)$, Eq. (5), one should notice that the positive V_2^0 corresponds to a negative cosmological constant $\Lambda=-V_2^0$ in GR if the term $\int V_2(\phi)\sqrt{-g}d^4x$ would appear in the GR action. For constructing models 1–3 of Sec. IV B, the positive definiteness of $V_2(\phi)$ (and therefore, the condition $V_2^0 > 0$) was one of the basic assumptions.

term $-\int \Phi\chi\Lambda_{eff}d^4x$ in the action would break this local symmetry. Thus, this additional, local symmetry can prevent a generation of the real cosmological constant by quantum corrections to TMT if no anomaly appears.

(b) Resolution of the flatness problem of the quintessential potential. The mechanism for the resolution of the flatness problem of the quintessence potential in TMT (question number two of the Introduction) is actually the same as the one used for the resolution of the cosmological constant problem. Since the TMT effective potential $U(\phi)$ takes a quintessence form as $\phi\to\infty$ due to the unbounded growth of the leading terms of the prepotentials V_1 and V_2, the appearance of any subleading terms (including terms generated by quantum corrections) in V_1 and V_2 cannot alter the shape of $U(\phi)$ as ϕ is large enough. There is no need for any of the coupling constants and mass parameters of the subleading terms to be very small. This is, in fact, the TMT answer to the question raised by Kolda and Lyth [11].

(c) Quintessential inflation type potential (also satisfying the cosmological nucleosynthesis constraint) obtained without fine-tuning. Two basic ideas have been used in this paper to demonstrate that TMT enables us to answer questions three and four of the Introduction. The fundamental role belongs to the first idea that, in the limit $\phi\to\infty$, the effective theory has to become invariant under shifts $\phi\to\phi+\text{const}$. A basis for this idea is the observation that if we want the effective theory to describe a quintessence as $\phi\to\infty$, the effective potential has to become flat as $\phi\to\infty$.

As it was shown by Guendelman [25,26], the role of the global continuous symmetries $\phi\to\phi+\text{const}$ in TMT belongs to transformations (18),(19) in the absence of fermions or, Eq. (38), in the presence of fermions. In terms of the dynamical variables used in the Einstein frame, these transformations are reduced to shifts of ϕ parametrized as in Eq. (19). In the models of Ref. [25], where the exponential form for the prepotentials, Eq. (16), with $\alpha=\beta$ being used, the global symmetry (18)–(19) is spontaneously broken. And although this symmetry is restored as $\phi\to\infty$, it is impossible in the framework of such a model to realize a quintessence scenario at $\phi>M_p$.

We have seen in the present paper that if a small explicit violation of the global continuous symmetries (18),(19) is present in the TMT original action (5) with the exponential form of the prepotentials, Eq. (16), then the TMT effective potential $U(\phi)$, Eq. (17), can be a suitable candidate for a quintessence model as $\beta\phi\gg M_p$. The smallness of the explicit symmetry breaking is formulated as a smallness of the dimensionless parameter $(\beta-\alpha)/\beta$ [see Eq. (22)].

In the absence of knowledge about the structure of the fundamental theory, and without any information about a mechanism leading to an explicit violation of the global continuous symmetry (18),(19), the quantity $(\beta-\alpha)/\beta$ is the only small parameter that can be used in attempts to modify the action with a simple exponential form of the prepotentials (16), with the aim to give rise to quintessential inflation type models. This can be done by adding terms that disappear as $(\beta-\alpha)/\beta$ tends to zero. This means that coupling constants in such additional terms have to be proportional to some positive power of this small parameter.

The second basic idea is that in the limit $(\beta-\alpha)/\beta\to 0$ (which leads us to the fundamental theory), the only mass parameter of the theory is the Planck mass M_p. This means that the dimensional coupling constants of the symmetry breaking terms have to be powers of the mass parameters m of the form $m=[(\beta-\alpha)/\beta]^n M_p,\ n>0$.

In the probe models studied in Sec. IV, we have chosen, just for illustration, $\beta=7,\ \alpha=6$, and hence, $(\beta-\alpha)/\beta=1/7$. Proceeding in the way described above, we reveal a remarkable feature of TMT: it is possible to achieve quite satisfactory quintessential inflation type models (see models 1–3 of Sec. IV) where, for the adjustment of the parameters, it is enough to use only mass parameters of a few orders less than M_p. We interpret this fact as the absence of a need for fine-tuning.

Besides the generation of the well-defined inflationary and quintessential regions of the TMT effective potential $U(\phi)$, one more remarkable result consists in the fact that the post-inflationary region of $U(\phi)$ has the exponential form $\propto\exp(-a\phi/M_p)$ with variable a, Eq. (31). This allows us to single out a region of $U(\phi)$, where a familiar approach [6] to a resolution of the problem with the cosmological nucleosynthesis constraint is realized without any additional assumptions.

(d) Resolution of the problems related to a possible direct coupling of the inflaton field to usual matter. As for question number five of the Introduction, the answer is quite clear: if the terms of the form $f_i(\phi/m)\mathcal{L}_i$, describing direct couplings of the inflaton field to the usual matter (see Ref. [19]), break the global continuous symmetry (38), they could appear in the original TMT action with small coefficients $f_i\propto[(\beta-\alpha)/\beta]^n,\ n>0$.

A direct coupling of the inflaton to fermionic matter is of a special interest. In the modified model studied in Sec. VI H, such a coupling enters the original action in the form of two Yukawa coupling type terms, Eqs. (39) and (66). The unbounded increase of V_1 and V_2 at the late universe works again in the desirable direction: the contributions of the Yukawa coupling type terms to the constraint (43) are negligible compared to V_1 and V_2. As we have seen, by adjustment of the parameters of the Yukawa coupling type interactions one can provide the presence of the direct coupling of fermionic matter to inflatons without observable effects at the late universe. The fermion mass then approaches constant and the correspondent long-range force disappears as $\phi\to\infty$.

It is worthwhile to notice here that the form of the Yukawa coupling type interactions, Eqs. (39) and (66), might be generalized without altering the results obtained for the late universe. In fact, if, for example, one modifies the interactions of the form $\propto\bar{\Psi}\Psi e^{\gamma\phi/M_p}\sqrt{-g}$ and $\bar{\Psi}\Psi e^{\tilde{\gamma}\phi/M_p}\Phi$ considered in Sec. VI H by adding the direct couplings of the form $\propto\bar{\Psi}\Psi\phi\sqrt{-g}$ and $\bar{\Psi}\Psi\phi\Phi$, respectively [which does not respect the global continuous symmetry, Eq. (38)], this has no effect on the late universe since the relative contributions of the adding terms are exponentially suppressed as $\phi>M_p$. At the early universe, for instance, as $\phi<0$, modifications like this could lead to observable effects. Such possibilities are additional tools given by TMT for adjustment of the field theory parameters to cosmological constraints of the early universe. One should stress that this is a merit of TMT, that *adjustment of the parameters determining the early universe evolution can be performed without any direct influence on the field theory parameters important for the late universe*.

(e) Background and matter fields quantization. There are some specific internal problems of TMT that we were forced to discuss in this paper. Fortunately it turns out that a resolution of those problems is closely related to the cosmological problems we were trying to solve here.

First of all, this is a problem of the definition of the gravitational background in TMT discussed in Sec. V and, related to this, a question of the choice between two large classes of the cosmological scenarios formulated in Sec. II. It turns out that only the second class of the cosmological scenarios (quintessential inflation belong just to this class) admits a satisfactory definition of the TMT gravitational background where the quantization of the inflaton field ϕ is a standard procedure.

The second TMT problem consists in the quantization of usual matter fields. In particular, fermionic field Ψ, in the model of Sec. VI, contributes to the constraint, Eq. (43), and hence, $1/\chi$ obtained by solving Eq. (43), will depend on $\bar{\Psi}\Psi$. In such a case, equations of motion in the Einstein frame, Eqs. (46),(48), and (49), would become very nonlinear. In Ref. [24], we have tried to avoid this sort of problem by starting from the original action that was very nonlinear in $\bar{\Psi}\Psi$.

In the present paper, where the inclusion of the usual matter is studied in the context of the models of Sec. IV B, and is intended to describe the quintessential-inflation scenario without fine-tuning, the problem of nonlinearity in matter fields does not appear. The reason is just due to a way that we solve the cosmological constant and other fine-tuning problems: the parameters of prepotentials $V_1(\phi)$, $V_2(\phi)$, and the integration constant M^4 are chosen such that the matter field contributions to the constraint, Eq. (43), are negligible compared to $V_1(\phi)$, $V_2(\phi)$, and M^4. Then, for $1/\chi$ we obtain the expression described by Eq. (53), the same as in the absence of the usual matter. As a result of this, in the Einstein frame the usual matter field equations in the background have canonical form, and their quantization becomes a standard procedure.

(f) SSB without generation of the cosmological constant. Reverting to the cosmological constant problem, it is worthwhile to notice in the conclusion that if the scalar (Higgs) field φ obtains a nonzero VEV, Eq. (47), the appearance of a constant part in $P(\varphi)$ just leads to a redefinition of m_1^4 [see Eq. (43)]. It is very important that in models 1–3 of Sec. IV B, m_1^4 has the order of $(10^{-2}M_p)^4$ or $(10^{-3}M_p)^4$. The correction we neglect in the left-hand side of Eq. (43), when replace it by Eq. (53), becomes of the order of $Q(\tilde{\varphi})/m_1^4$, where Q is a polynomial in $\tilde{\varphi}$ ($|\tilde{\varphi}|\ll v\lesssim m_1$) that satisfies the condition $Q(0)=0$. Thus, if $|P(v)|<m_1^4$, then spontaneous breaking of a gauge symmetry does not affect the magnitude of the effective cosmological constant (at the late universe) imitated by the quintessential potential, Eq. (29).

Another possibility appears if the whole term $m_1^4 e^{\alpha\phi/M_p}$ in the prepotential $V_1(\phi)$ is generated by SSB. In such a case the quintessential potential becomes

$$U(\phi)\approx\frac{[P(v)]^8}{M_p^4}e^{-2(\beta-\alpha)\phi/M_p}. \tag{73}$$

This is the TMT mechanism which, together with the shape of $U(\phi)$ in the inflationary region predicted by each of the models 1–3 of Sec. IV B, provides a resolution of one of the most serious aspect of the cosmological constant problem [10]: the need for an enormous fine-tuning of initial conditions in models with SSB in order to satisfy the dual requirement of large Λ in the past and small Λ at present.

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